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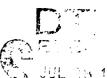
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MULTI-VALUED COMPONENT SYSTEMS

FINAL REPORT

Air Force Office of Scientific Research (AFSC)
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ABSTRACT

A large number of reliability models have been considered under the AFOSR Grant AFOSR 77-3213.

Our main goal on this project has been to develop theory and models for analyzing reliability systems.

One area of interest was in analyzing multi-component systems. In [1], Ross considered a reliability system composed of n components each of which is operating at some performance level. He supposed the existence of a nondecreasing function ϕ, called the structure function, such that $\phi(x_1, \ldots, x_n)$ denotes the performance level of the system when the ith component's performance level is x_i, $i = 1, \ldots, n$. He allowed both x_i and $\phi(x_1, \ldots, x_n)$ to be arbitrary nonnegative numbers and extended many of the important results of the usual binary model to this more general framework. In particular he obtained fundamental inequality for $E[\phi(X_1, \ldots, X_n)]$ when ϕ is binary which was used to generate a host of inequalities concerning IFRA distributions including, as a special case, the IFRA convolution theorem. He also defined the concept of an IFRA stochastic process and proved the analog of the IFRA closure theorem.

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In a follow up paper, Ross [2] used the results of [1] to generalize the usual Poisson shock model. Specifically, he supposed that shocks hit a device in accordance with a non-homogeneous Poisson process with intensity function $\lambda(t)$. The $i\underline{th}$ shock causes a damage X_i. The X_i are assumed to be independent and identically distributed positive random variables, and are also assumed independent of the counting process of shocks. Let $D(x_1, \ldots, x_n)$ denote the total damage when n shocks having damages $x_1, \ldots, x_n$ have occurred. It had previously been shown that the first time that $D(\underline{X})$ exceeds a critical threshold value is an increasing, failure rate average random variable whenever (i) $\lambda(t) = \lambda$ and (ii) $D(\underline{x}) = \sum x_i$. He extended this result to the case where $\int_0^t \lambda(s)ds/t$ is nondecreasing in t and $D(\underline{x})$ is a symmetric, nondecreasing function.

In [3], Ross, Shahshahani and Weiss considered the usual n component model in which either component is at any time either on or off. They supposed that the $i\underline{th}$ component is initially on and stays on for a random time T_i at which point it goes off and remains off forever. The random times T_i, $i = 1, \ldots, n$ were assumed to be independent and identically distributed continuous random variables. They studied the properties of N, the number of components that are off at the moment the system goes off. They then (i) computed the factorial moments of N in terms of the reliability function; (ii) proved that N is an increasing failure rate average random variable. They then

considered the special structure in which the minimal cut sets do not overlap and proved that N is an increasing failure rate random variable.

In [4], Ross along with Shahshahani and Weiss considered an r-player version of the famous problem of the points. At each play of a game, exactly one of the players wins a point--player i winning with probability p_i . The game ends the first time a player has accumulated his required number of points--this requirement being n_i for player i . Their main result was to show that N , the total number of plays, is an increasing failure rate random variable. In addition, they proved some Schur convexity results regarding $P\{N \leq k\}$ as a function $\underline{p}$ (for $n_i \equiv n$) and as a function of $\underline{n}$ (for $p_i \equiv 1/r$) .

In [5], Brown and Ross introduced the concept of the observed hazard rate. If X is the life of some system, then they defined the observed hazard rate at time t , call it R(t) , as the instantaneous probability (density) of failure of X at time t given survival up to t and given a complete description of the system state at t . They conjectured that the total observed hazard--namely, $\int_0^X R(t)dt$--is an exponential random variable with mean 1 and verified this for the special case when X is the distribution of system life of an n component system having an arbitrary monotone structure function.

In [6], Derman, Ross, and Schechner gave a simple probabilistic proof that starting in state 0 the first passage time

to state n is, for a birth and death process, an IFR random variable.

In [7], Ross and Schechtman considered a system consisting of n separately maintained independent components where the components alternate between intervals in which they are "up" and in which they are "down." When the $i\frac{\text{th}}{}$ component goes up [down] then, independent of the past, it remains up [down] for a random length of time having distribution $F_i[G_i]$ and then goes down [up]. We say that a component is failed if it is down and has been down for the previous A time units. Assuming that all components initially start "up", let T denote the first time they are all failed, at which point we say the system is failed. Properties of T were obtained.

In [8], Schechner studied the conditional distribution of $X(t)$ given $\sup_{0 \le u \le t} X(u) < a$ of certain processes. He used it to prove the IFR property of k-out-of-n systems and certain shock models and other processes.

In [9], Schechner supposed than an N-component parallel system is subjected to a known load program. As time passes, components fail in a random manner which depends on their individual load histories. At any time t , the surviving components share the total load according to some rule. The system's lifetime distribution is studied under various breakdown rules. Under the linear breakdown rule, it is shown that if the load program is increasing the system lifetime is IFR. Using the notion of Schur convexity, stochastic comparison of different

system is obtained. It is also shown that the system failure time is asymptotically normally distributed as the number of components grows large. All these results hold under various load sharing rules, in fact, one can prove that the system lifetime distribution is invariant under different load sharing rules.

In [10], Derman, Lieberman and Ross analyzed the consecutive k-of-n system in which there are n components linearly ordered. Each component either functions or fails and the system is said to be failed if any k consecutive components are failed. Let $r(\underline{p}) = r(p_1, \ldots, p_n)$ denote the probability that the system does not fail given that the components are independent, component i functions with probability p_i , $i = 1, \ldots, n$. The function $r(\underline{p})$ is called the reliability function. They studied this system both when the components are linearly ordered and also when they are arranged in a circular order. They consider the case where all p_i are indentical and present a recursion for obtaining the reliability of a consecutive k-of-n in terms of the reliability of a consecutive $k - 1$ of n system. This yields simple explicit formulas when k is small. They showed how upper and lower bounds on $r(\underline{p})$ can be simply obtained. They then considered a dynamic version in which each component independently functions for random time having distribution F , and showed that when F is increasing failure rate (IFR), then system lifetime is also IFR only in the circular case when $k = 2$. They considered a sequential optimization model in the linear $k = 2$ case. In this model, components are put in place one at a time with complete

knowledge as to whether the previous component has worked or not. We show that the optimal policy is such that whenever a success occurs we follow it with the worst of the remaining components and whenever a failure occurs we follow it with the best of the remainder.

In [11], Derman, Lieberman and Ross considered an N server queuing system in which service times of server i are exponentially distributed random variables with rate λ_i. Customers arrive in accordance with some arbitrary arrival process. If a customer arrives when all servers are busy, then he is lost to the system; otherwise, he is assigned to one of the free servers according to some policy. Once a customer is assigned to a server he remains in that status until service is completed. They showed that the policy that always assigns an arrival to that free server whose service rate is largest (smallest) stochastically minimizes (maximizes) the number in the system. The result was then used to show that in an N component system in which the ith component's up-time is exponential with rate λ_i and in which the repair times are exponential with rate μ, the policy of always repairing the failed components whose failure rate λ is smallest stochastically maximizes the number of working components.

Other optimization models supported by the grant were those presented by Pinedo and Ross in [12] and by Weiss and Pinedo in [13], and by Kan and Ross [14].

Other models supported by the grant were those of Ross in [15] dealing with random graphs; and Ross [16] and Fond and Ross [17] dealing with queues with nonstationary arrival processes.

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